Valuation: Volatility Surfaces

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Volatility is a measure of the uncertainty regarding the magnitude of change in the future value of a financial instrument. It is a key parameter in the pricing of options. All other things being equal, high volatility implies a higher option price and low volatility a lower price. Basic option pricing models often assume that volatility is constant and independent of both an option's strike price and its time-to-expiry. In practice this rarely occurs so, for correct option valuation, we need to capture the interrelationship between volatility, strike price and time-to-expiry through a *volatility smile* or a *volatility surface*.

Estimating Volatility

Volatility is not easily observable in the market and has to be estimated either by:

- Calculating the standard deviation of the relative price movements over time or
- Calculating the *implied volatility* according to a pricing model, i.e. back-solving the model to calculate the volatility such that the calculated model value of the option is equal to the observed price of the option in the market.

Practitioners tend to prefer the second method since it can be viewed as a market participant's assessment of volatility. However, this method is dependent on the underlying price process and any other assumptions made by the model (for example, the Black-Scholes model assumes that there are no transaction costs). Specifically, basic option valuation models assume that:

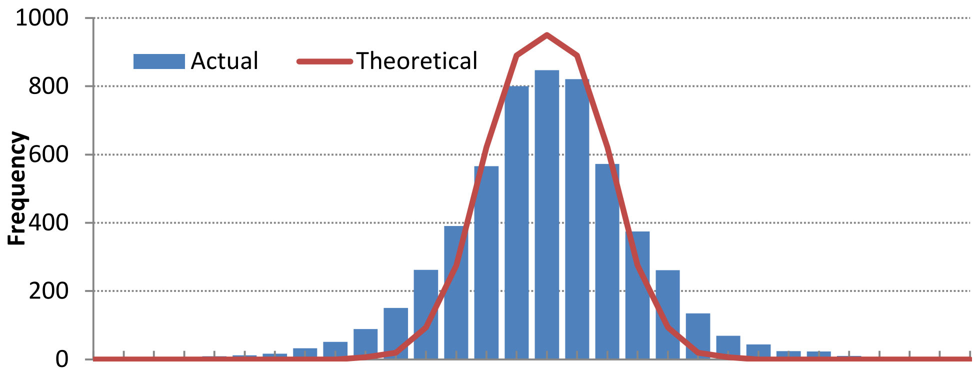
- The volatility of the underlying is constant and
- Changes in the value of the underlying are smooth, i.e. without any jumps. This implies that the distribution of returns is lognormal, i.e. the log returns are assumed to fall within a standard normal curve.

If the above two assumptions were true, then it would follow that all options on the underlying instrument would yield the same implied volatility. In practice, however, this theoretical idealisation is rarely observed and evidence has shown that volatilities tend to differ across both the strike price of the option and the time-to-expiry.

Volatility Smile

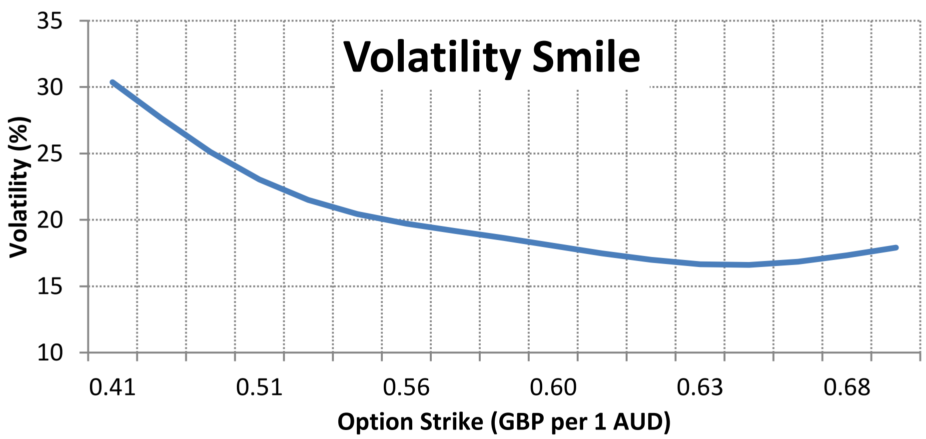
The market observation which suggests that volatility is a function of strike price is known as a volatility smile. It means that the implied volatility of at-the-money options is usually lower than the volatility of deep-in-the-money or deep-out-of-the-money options. **Figure 1** below shows the distribution of the log returns of the AUD-GBP exchange rate over a 21-year period from 1990 to 2011. Overlaying the histogram is the theoretical normal distribution with the same mean and standard deviation as the returns on the AUD-GBP exchange rates. As can be seen from **Figure 1**, the distribution of historical returns shows a higher probability of extreme movements in the exchange rates than predicted by the theoretical model (these are the points where the blue bars of the histogram extend above the red curve of the normal distribution). Such a distribution is called a *fat-tailed* distribution and has important ramifications for the valuation of options which are either *deep-in-the-money* or *deep-out-of-the-money* (options which have a strike quite far from the current spot).

Figure 1: Histogram showing the daily log returns of the AUD-GBP exchange rate over a 21-year period from 1990 to 2011. Overlaid on the histogram is the theoretical normal distribution with the same mean and standard deviation as the histogram (Data sourced from Thomson Reuters)



As option valuation models are probabilistic, the value of an option depends upon the probability of exercise. A high volatility increases the probability of large movements and thereby increases the chance of exercise. It follows that models which show a high probability of large movements in the underlying will calculate a high implied volatility. Call options for example, give the holder the right to buy the underlying and have a payoff if the underlying is above the strike price. A deep-out-of-the-money call option has a strike rate which is quite high in comparison to the spot rate. Under a lognormal model, the probability of exercise of such an option is quite low and therefore the option will be under-valued or, in other words, the implied volatility of an option derived from the lognormal model will be lower than expected. Similarly, the lognormal model would imply a higher volatility for at-the-money options since it overestimates their probability of exercise.

Figure 2: Volatility Smile - the relationship between the strike price and the volatility observed for AUD-GBP

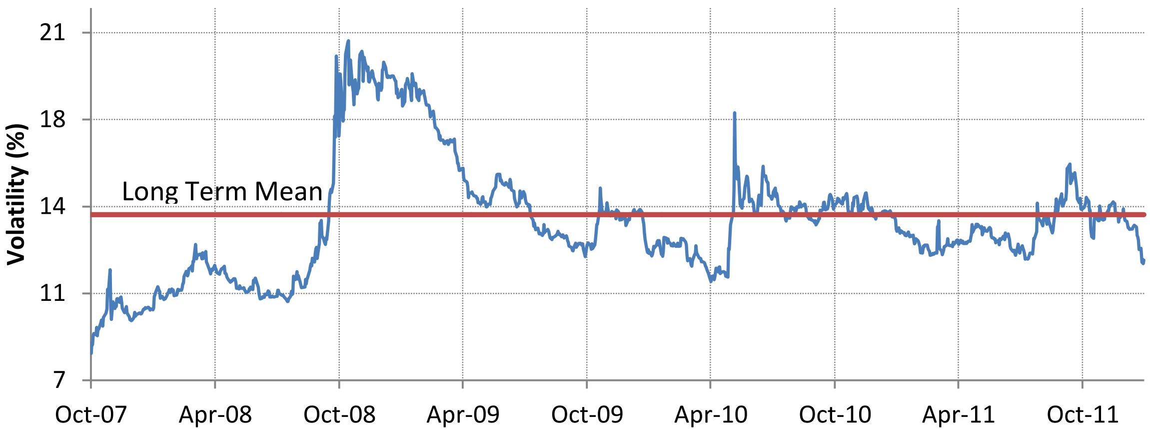


Term Structure of Volatility

In addition to the strike price, volatility is also a function of the time-to-expiry of an option. Trend analysis of historic volatility data has shown that volatility tends to be mean reverting, i.e. given sufficient time; volatility tends to revert to its equilibrium level or long-term average.

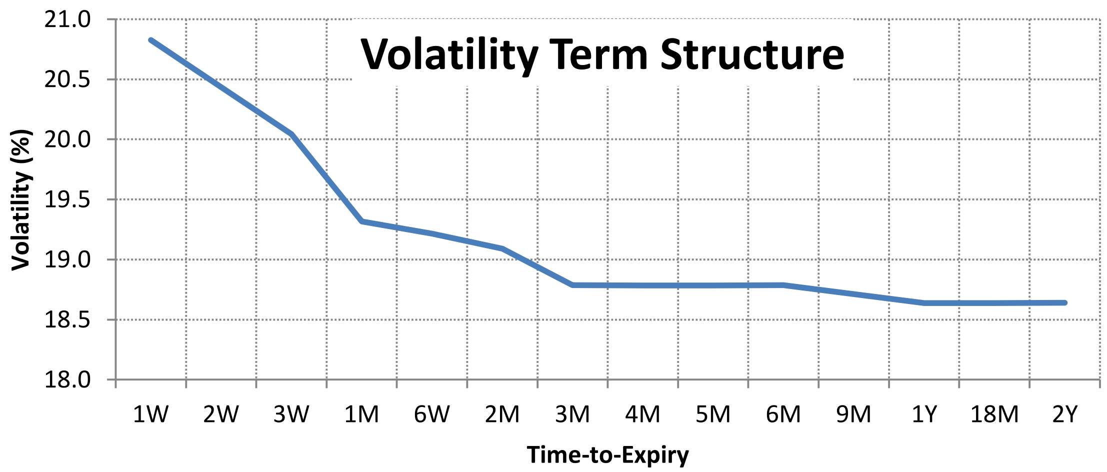
Figure 3 below shows the historic one-year at-the-money volatility for AUD-GBP exchange rate. The long-term mean of the data series is approximately 14%. As can be seen from the figure, despite some spikes, volatility tends to revert to its long-term equilibrium value over time.

Figure 3: AUD-GBP one-year volatility over the four-year period 2007-2011 showing mean reversion around the long-term mean of 14%



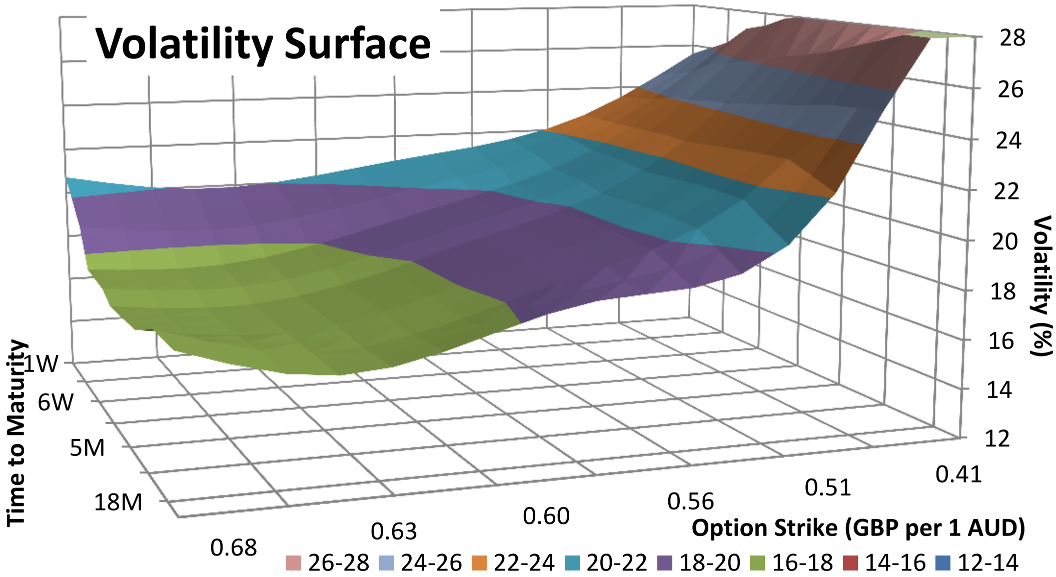
Hence, if the current volatility levels are historically low, then the expectation is that volatility will increase over time and as such the volatility used for valuing an option should be an increasing function of its time-to-expiry. On the other hand, if the current volatility levels are historically high, then the expectation is that volatility will reduce over time and as such the volatility for an option should be a decreasing function of its time-to-expiry.

Figure 4: Variation of at-the-money volatility over time for AUD-GBP exchange rate



A combination of the effects of both the time-to-expiry and the strike of an option on the volatility is called the v*olatility surface* – normally referred to as the *term structure of the volatility smile*. Both the effects of time-to-expiry and strike rate have to be taken into account for accurate option valuations.

Figure 5: Volatility Smile – the relationship between the option volatility, time-to-expiry and strike price for AUD-GBP exchange rate



As an example, let us consider a 1 year call option to sell GBP 10.0M at 0.4861 GBP per AUD. The one-year *at-the-money* volatility for AUD-GBP exchange rate is 18.71% and at-the-money-strike is calculated to be 0.5822. Using the at-the-money volatility the market value of such an option is calculated to be approximately AUD 4.0M. However, if we consider the volatility smile, then the implied volatility for such an option is actually 25.12% - significantly higher than the at-the-money volatility, and the market value is calculated to be AUD 4.35M. It can be seen that ignoring the volatility smile in the valuation of such a deep-in-the-money option significantly undervalues the option.

The volatility of any underlying instrument is not easily observable in the market and often has to be inferred by the observable price of the traded options. Implied volatilities calculated from observed market prices show a significant *smile whereby deep-in-the-money* and *deep-out-of-the-money* options have significantly higher volatilities than *at-the-money* options. Because of this, valuation models need to incorporate the effect of such a smile or run the risk of significantly undervaluing options.