

Under the Hood - Option Valuation

John Clarson

This valuation article will focus on the *valuation of options*. It will provide an overview of some different methodologies that can be applied to option valuation as well as discuss the basic principles of volatility.

Option Valuation - a Basic Example

Suppose Bank A sells an Australian exporter an option to **sell** USD750,000 and **buy** AUD1,000,000 at a rate of 0.7500 in one month's time. The current exchange rate is 0.7506 and the current one month forward rate is 0.7500. How could the exporter go about valuing this option in the absence of a standard formula or any other information?

Step 1: Estimate a range of possible outcomes for the exchange rate and assign a probability to each outcome.

Let's say that the exporter decides on the following range below. Note that most of the likely outcomes are based around where the market currently is, i.e. one month forward rate of 0.7500.

Range	Mid	Probability	AUD Value	AUD Weighted Value
0.7100-0.7200	0.7150	5%	0	0
0.7200-0.7300	0.7250	8%	0	0
0.7300-0.7400	0.7350	14%	0	0
0.7400-0.7500	0.7450	23%	0	0
0.7500-0.7600	0.7550	23%	6,623	1,523
0.7600-0.7700	0.7650	14%	19,608	2,745
0.7700-0.7800	0.7750	8%	32,258	2,581
0.7800-0.7900	0.7850	5%	44,586	2,229
Total		100%		9,078

Step 2: Calculate the value of the option at maturity for each of these outcomes and weight them based on the assumed probability.

For example, if spot ends up between 0.7700 and 0.7800 (average 0.7750) then the average gain from buying AUD1m at the strike of 0.7500 will be AUD32,258 ($USD750,000 \times (1/0.7500 - 1/0.7750)$). The chances of this happening are 8% which means this equates to a positive value for the option holder of AUD2,581 ($AUD32,258 \times 8\%$).

Based on the assumptions, the exporter would decide that the option is worth AUD9,078 or about 0.9% of the notional principal. This is actually the future value of the option, so the exporter would also have to discount this back to its present value.



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The Underlying Theory

While the above example is a simplification, it shows the general method for valuing options:

- a. Determine a distribution of outcomes for the underlying instrument, then
- b. Calculate a weighted average of the terminal value of the option.

However, building an entire distribution from first principles and doing these calculations is not really practicable. The **Black-Scholes** formula (dating from 1973) made an underlying assumption on how prices of financial instruments are distributed. They assumed that the distribution was log-normal meaning that the daily percentage move of the instrument is normally distributed. Using the lognormal property of returns, it is possible to derive a formula for calculating the fair value of options. The inputs required for valuation using this formula are:

1. Strike price (0.7500)
2. Spot price (0.7506)
3. Time to maturity one month (0.0833 years)
4. Foreign currency interest rate and domestic interest rate (AUD:6.25% USD:5.25%)
5. Volatility (9%)

For the option in our example using the Black-Scholes formula along with the above stated inputs gives a value of AUD10,416. Note the domestic interest rate used is the USD rate because the *price* (exchange rate) is denominated in USD.

Similar valuation methods and formulae can be used for interest rate options and interest rate swaptions. For example, the formula for a cap is derived from a distribution of the short term interest rate over the life of the cap. Each individual rate set (or caplet) is valued separately based on the potential payoff where the short term rate exceeds the strike on the reset date. In this case, the spot price is replaced by the forward interest rate and the only other interest rate required is that used to discount the option values back to today.

For commodity options, the foreign currency interest rate is termed the 'convenience yield' and is the interest rate such that combined with the underlying currency rate gives the current forward curve. It is easier to use the forward (or futures) price directly rather than the spot. Now the convenience yield is not required and only the base currency interest rate is needed.

Volatility + Black Scholes

Volatility is the standard deviation (usually quoted on an annualised basis) of the percentage returns of an instrument or in other words it is the uncertainty about the returns provided by the instrument. Volatility, in combination with the time to maturity, determines how widely spread the possible price outcomes are. The wider the spread the greater the value of the option as can be seen from the simple example above. This means that the value of an option increases with time to maturity and volatility.

In our example we used a volatility of 9%. This means that in one year, we expect that 95% of the time the exchange rate will be in a range of $\pm 18\%$. Volatility can be scaled for different periods so a vol of 9% for one month means that the price will move in a range of $\pm 5.2\%$ ($18\%/\sqrt{12}$). This roughly corresponds to the estimated range of our exporter which is why our approximate valuation corresponds with the theoretical valuation (AUD9,078).

At-the-money volatility is often available for common exchange rates, interest rates and commodities. Using this volatility for the relevant time period and using a Black-Scholes option valuation tool (these are commonly available) will give you a reasonable estimate of the value of an option.

However using at-the-money vol may be inaccurate in some cases. Depending on the currency or interest rate, the volatility can vary with the strike price. For example, when USD interest rates were at 1%, the volatility used for valuation of a cap at 3% would be quite different to that used for valuation of a cap at 5%. The reason volatility varies like this is that the lognormal distribution is only an approximation of how prices vary over time. To overcome this problem, market makers will quote different volatilities for different strike prices. It is these varying volatilities that can be difficult for indirect participants to determine.



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www.visualrisk.com

Phone: +61 2 9262 6969
E-mail: sales@visualrisk.com

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